

Exam. Code : 211002

Subject Code : 5543

M.Sc. (Mathematics) 2<sup>nd</sup> Semester

MECHANICS—II

Paper—MATH-564

Time Allowed—3 Hours]

[Maximum Marks—100

**Note** :— Attempt ten questions in all, selecting **TWO** questions from each unit. All questions carry equal marks.

## UNIT—I

I. Prove that :

$$\vec{V}_p = \vec{V}_0 + \vec{\omega} \times \vec{r}$$

where symbols have usual meaning.

II. Discuss kinematics of rigid lithospheric plate motions on a rotating earth.

III. A force  $\vec{F}$  acts on a particle constrained to move along a curve C joining points A and B. Prove that work done is :

$$W = \int_{t_A}^{t_B} A dt, A \text{ being power.}$$

IV. Find  $\vec{I} = m\vec{v}_2 - m\vec{v}_1$  and then show that if the velocity of a particle of mass m changes from  $\vec{v}_1$  to  $\vec{v}_2$  due to impulse  $\vec{I}$ , then K.E. gained is  $\frac{1}{2} \vec{I} \cdot (\vec{v}_2 + \vec{v}_1)$ .

## UNIT—II

V. Prove that :

$$A^2 \omega_1^2 + B^2 \omega_2^2 + C^2 \omega_3^2 = \text{Const},$$

$\omega_1, \omega_2, \omega_3$  are the angular velocities along and A, B, C are the principal moments of inertia about the axes.

VI. Prove that the motion of a rigid body about a fixed point may be represented by the rolling of an ellipsoid fixed in the body upon a plane fixed in space.

VII. A uniform solid sphere rolls without slipping on a rough horizontal plane which is rotating with uniform angular velocity about a vertical axis. If there are no force acting on the sphere save its weight and the friction at the contact, prove that the focus of the centre of the sphere is a circle.

VIII. A uniform rectangular lamina of sides  $2a, 3a$  is freely hinged to a horizontal axis along one of its shorter edges. This axis is fixed to a vertical shaft which passes through the midpoint of the hinged edge, and the shaft is forced to rotate with constant angular velocity  $\omega$ . If  $\theta$  is the inclination of the plate to the downward vertical at time  $t$ , show that Euler's dynamical equations can be written

$$\text{as : } a \ddot{\theta} - a \omega^2 \sin \theta \cos \theta = -\frac{1}{2} g \sin \theta.$$

## UNIT—III

IX. Prove that :

$$\frac{d}{dt} \left( \frac{\partial T}{\partial \dot{q}_j} \right) - \frac{\partial T}{\partial q_j} = Q_j \quad (j = 1, 2, \dots, n)$$

in usual notations.

- X. Discuss equilibrium configurations for conservative holonomic dynamical systems.
- XI. Determine virtual work function for flyball governor.
- XII. Explain normal periods of oscillation.

## UNIT—IV

XIII. Prove that the necessary and sufficient condition for

$$\int_{x_1}^{x_2} f(x, y, y') dx \text{ to be an extremum is that :}$$

$$\frac{\partial f}{\partial y} - \frac{d}{dx} \left( \frac{\partial f}{\partial y'} \right) = 0.$$

- XIV. Describe Hamilton's principle.
- XV. A particle of mass  $m$  moves on  $xy$ -plane under the influence of a force of attraction to the origin of magnitude  $F(r) > 0$ , where  $r$  is the distance of the mass from the origin. Find the Lagrange's equation of motion.
- XVI. Explain extension of the variational method.

## UNIT—V

XVII. Find the extremals of the functional

$$\int_0^{\frac{\pi}{2}} \left\{ 2xy + \left( \frac{dx}{dt} \right)^2 + \left( \frac{dy}{dt} \right)^2 \right\} dt .$$

XVIII. Find the extremals of the functional :

$$I[y(x)] = \int_{x_0}^{x_1} (2xy + (y''')^2) dx .$$

XIX. Explain Geodesics.

XX. Explain Galerkin's method.